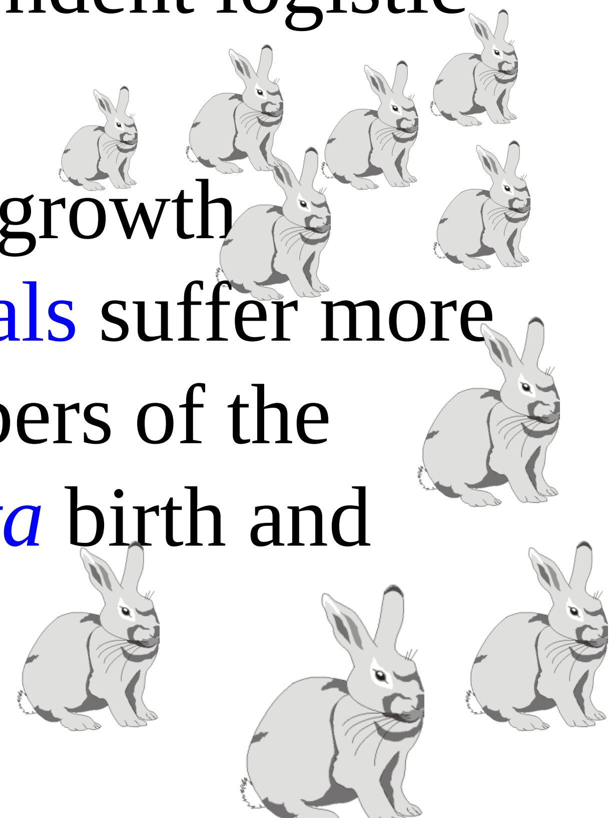
Lotka-Volterra Competition

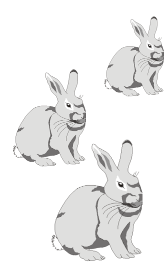
Intraspecific Competition

- Intraspecific (within species) competition is described by the density-dependent logistic growth model.
- As a population with logistic growth increases in density, individuals suffer more competition from other members of the population and their *per capita* birth and survival rates decline.



Interspecific Competition

- The process of interspecific (between-species) competition is directly analogous to processes of intraspecific competition.
- The Lotka-Volterra model incorporates competition within and between species.



Possible Outcomes of Competitive Interaction

- Coexistence
- Exclusion of the inferior competitor

One objective for the Lotka-Volterra competition model is to predict what conditions lead to one or the other outcome.

Lotka-Volterra is Extension of Continuous Logistic Growth Model

Recall the logistic growth model equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(\frac{K - N}{K}\right)$$

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = r_1 N_1\left(\frac{K_1 - \left(N_1 + \alpha N_2\right)}{K_1}\right)$$

The Lotka-Volterra model uses subscripts to distinguish two competing species.

This incorporates both intra- and interspecific effects to the equation.

Parameters

$\boldsymbol{N_1}$ The density of species 1

$\boldsymbol{N_2}$ The density of species 2

$r_{\mathbf{1}}$ (births – deaths) of species 1

$K_{\mathbf{1}}$ Carrying capacity of species 1

♋ Scaling factor:

Since species 1 and 2 have different biological properties, they are unlikely to make exactly equivalent demands on the habitat. The scaling factor accounts for these differences.

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = r_1 N_1 \left(\frac{K_1 - \left(N_1 + \alpha N_2\right)}{K_1} \right)$$

More About ♋

If species 1 is a bird that eats mostly seeds, and a few insects while its competitor, species 2, eats mostly insects and a few seeds, then each interspecific competitor will cause less interference than an intraspecific competitor and

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Competition Coefficient

- The scaling factor, ♋📭 is called the competition coefficient.
- It quantifies the *per capita* reduction in population size of species 1 caused by species 2.

Analogous Equation for Species 2

- Since competition is an interaction between two species, there is an analogous equation giving the dynamics of species 2:

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = r_2 N_2 \left(\frac{K_2 - \left(N_2 + \beta N_1 \right)}{K_2} \right)$$

Equation for Species 2 cont.

Since there is no reason to believe that the reciprocal interference of two species will be precisely equal, a second competition coefficient, β, is defined.

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = r_2 N_2 \left(\frac{K_2 - \left(N_2 + \beta N_1 \right)}{K_2} \right)$$

β quantifies the per capita displacement of species 2 by species 1.

Lotka-Volterra Equations

Together, these equations are the Lotka-Volterra competition equations, named in honor of two ecologists who proposed them independently (Lotka 1925, 1932; Volterra 1926).

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = r_1 N_1 \left(\frac{K_1 - \left(N_1 + \alpha N_2\right)}{K_1} \right)$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = r_2 N_2 \left(\frac{K_2 - \left(N_2 + \beta N_1\right)}{K_2} \right)$$

Dynamics of Lotka-Volterra Competition

- Deriving an equation to project future populations cannot be accomplished in the same way as for the exponential or logistic growth equations.