Density-Independent Population Growth

Density Independent

- Means that the rate of population growth *per capita*, or per individual, is independent of density.
- Density is the population size per unit area
- Individuals do not interfere with the reproduction, development, or survival of their neighbors.

Exponential Growth with Continuous Breeding

Populations

- Increase through:
  - Births
  - Immigration
- Decrease through:
  - Deaths
  - Emigration

Assumptions

- Immigration and emigration cancel
- Population is increased and decreased only through births and deaths
- All individuals are equal in terms of probability of dying or producing offspring
- The population exists entirely of **parthenogenetic** females (asexual reproduction)
- Environmental resources are infinite

Assumptions (cont.)

- Organisms like Homo sapiens or the bacteria in a culture flask, with continuous breeding and overlapping generations.
- All ages will be present simultaneously
- Population size will change steadily in small increments with the birth and death of individuals at any time.

Continuous Population Growth

- **Continuous** population growth is best described by a *differential * equation*

  *differential: the change of one thing in respect to the change of something else.

change in the size of the population

with respect to a change in time.

$$\frac{\Delta N}{\Delta t}$$

Parameters

$N(t)$	is population size: a continuous function of time, t
b	is the instantaneous birth rate per individual
d	is the instantaneous death rate per individual

Equation

$$\frac{dN}{dt} = (b - d)N$$

The rate of change of population (N) with respect to time (t)

Births – deaths multiplied by population

$(b - d)$

- If births – deaths is positive (births > deaths), the equation $\frac{\mathrm{d}N}{\mathrm{d}t} = (b - d)N$ will be positive and the population size will increase
- Likewise, if $(b - d)$ is negative, the change in the population size will decrease.

Intrinsic Rate of Increase (r)

- Also called the instantaneous growth rate

$$r = (b - d)$$

The equation becomes ➡

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN$$

The Effect of *r* on the Equation

- When $r = 0$, birth and death rates balance, individuals just manage to replace themselves
- Population size remains constant.
- When $r < 0$, the population shrinks toward extinction
- When $r > 0$, it grows.

To Project Future Population Sizes

- Integrate the equation $\frac{\mathrm{d}N}{\mathrm{d}t} = rN$
- Although r is an instantaneous rate, its numerical value is only defined over a finite interval. If this rate remains constant, then we can predict future population size, N(t) from a knowledge of the constant growth rate (r), the present population size, N(0), and the time over which growth occurs (t).

Per capita Population Growth Rate

- Divide the whole population-growth rate, dN/dt, by population size, N, to get growth rate *per capita* (per individual).

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}t} = r\cancel{N}\frac{1}{\cancel{N}} \quad \Rightarrow \quad \frac{\mathrm{d}N}{N\mathrm{d}t} = r$$

To Get Population Size as a Function of Time: Integrate

- Rearrange to get… $\mathrm{d}N = rN\,\mathrm{d}t$
- Rearrange again… $$\frac{\mathrm{d}N}{N} = r\,\mathrm{d}t$$
- Integrate… $$\int_{t_0}^{t} \frac{\mathrm{d}N}{N} = \int_{t_0}^{t} r\,\mathrm{d}t$$
- gives ➡ $\ln N(t) - \ln N(t_0) = rt - rt_0$ ➡

Rearrange Again Using *e*

- Use the natural logarithm *e* to get

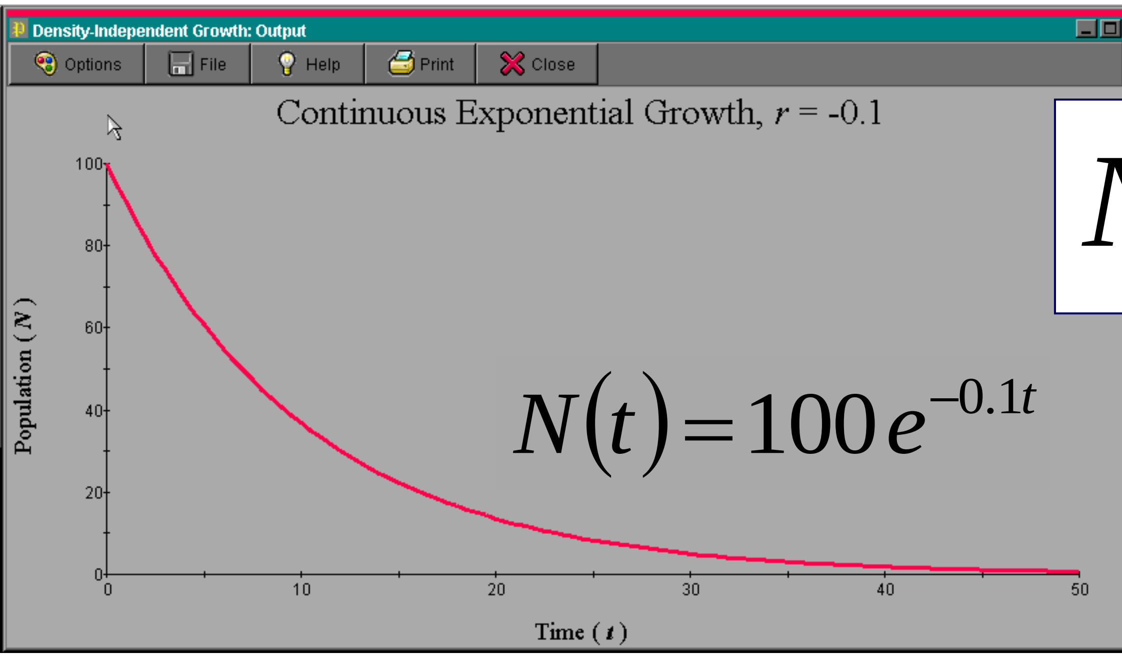
$$\ln N(t) - \ln N(t_0) = rt - rt_0$$

$$\Rightarrow \quad e^{\ln N(t) - \ln N(t_0)} = e^{rt - rt_0}$$

$$\Rightarrow \quad \frac{N(t)}{N(t_0)} = e^{rt - rt_0} \quad \Rightarrow \quad \text{when } t_0 = 0$$

$$N(t) = N(t_0)e^{rt}$$

Equation to Predict Future Population Size



$$N(t) = N(t_0)e^{rt}$$

Equation allows projection of future population size $N(t)$, from a knowledge of the constant growth rate, r, the present population size, $N(t_0)$, and the number of intervals, t, over which growth occurs.

Example Exponential Growth Problem

- If an exponentially growing population increases from $N = 5$ to $N = 25$ in one week, how long will it take to increase from 25,000 to 125,000?

Problem 1 (cont.)

- If an exponentially growing population increases from $N = 5$ to $N = 25$ in one week, how long will it take to increase from 25,000 to 125,000?

$t = 7$

$$N(t) = N(t_0)e^{r7}$$

Problem 1 (cont.)

- If an exponentially growing population increases from $N = 5$ to $N = 25$ in one week, how long will it take to increase from 25,000 to 125,000?

$N(t) = 25$

$N(t_0) = 5$

$$25 = 5e^{r7}$$

Problem 1 (cont.)

$$\frac{25}{5} = \frac{5}{5}e^{r7} \Rightarrow 5 = e^{r7} \Rightarrow \ln(5) = \ln\left(e^{r7}\right)$$

$$\Rightarrow \ln(5) = r7 \Rightarrow$$

$$\frac{\ln(5)}{7} = r$$

Problem 1 (cont.)

- If an exponentially growing population increases from $N = 5$ to $N = 25$ in one week, how long will it take to increase from 25,000 to 125,000?

Now that you know r, you can solve for t in the same way using $N(t) = 125{,}000$ and

$N(t_0) = 25{,}000$

Problem 2 (For Fun)

- The population of the U.S. in 1990 was: **248,709,873**
- The population of the U.S. in 2000 was: **281,421,906**

What is the expected population in 2010?

Data source: www.census.gov

Problem 2 (For Fun, cont.)

$$281421906 = 248709873e^{r10}$$

$$\frac{281421906}{248709873} = e^{r10}$$

$$\ln\left(\frac{281421906}{248709873}\right) = r10 \Rightarrow \frac{.1236}{10} \approx r$$

$$\Rightarrow 0.0124 \approx r$$

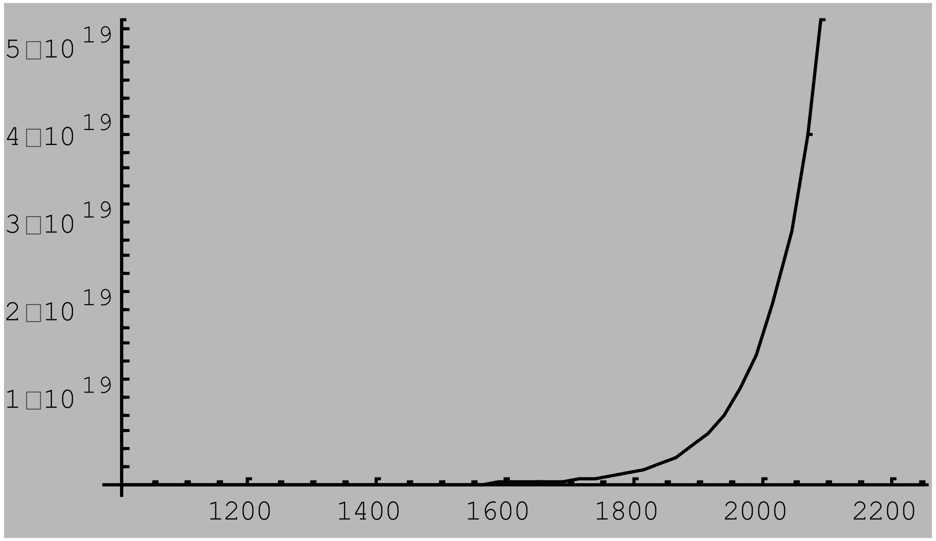
Problem 2 (For Fun, cont.)

- What will the population be in 2010?

$$N(t) = 281{,}421{,}906\ e^{(.0124)10}$$

I get
318,574,064

Try graphing
the equation ➡

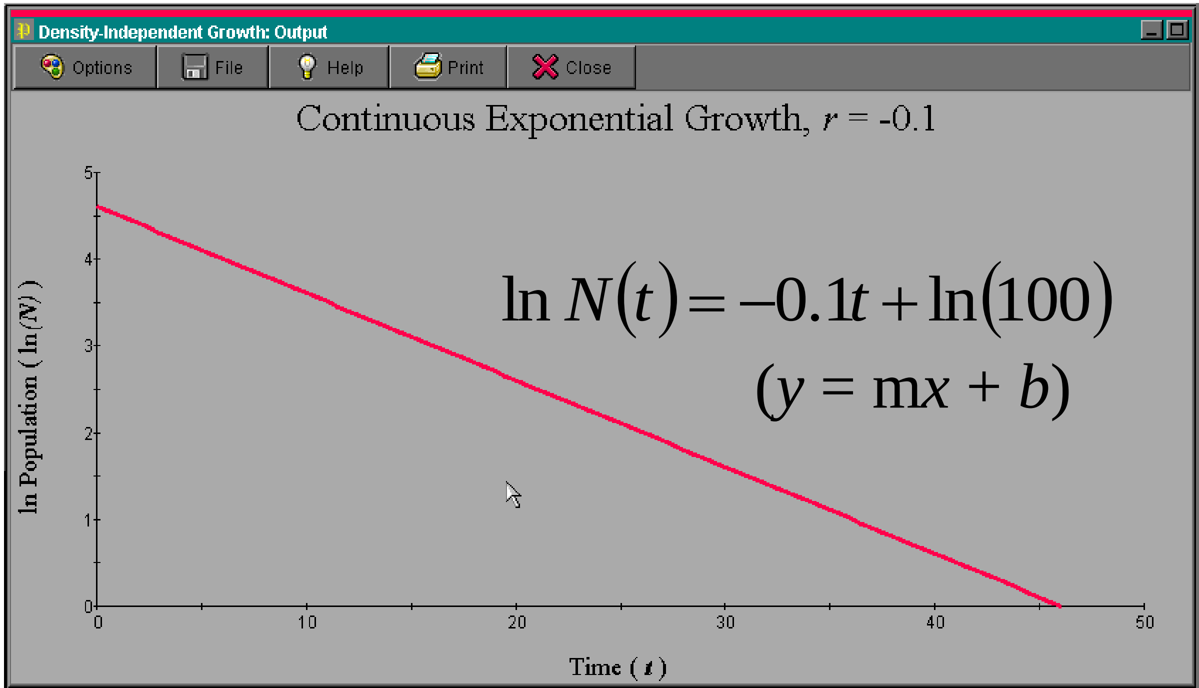


Mathematica graph

The Natural Logarithm Plot

- Take the natural log of both sides of the equation:

$$\ln N(t) = \ln N(t_0) + \ln e^{rt} \Rightarrow \ln N(t) = \ln N(t_0) + rt$$



Geometric Growth of Discrete Cohorts

Discrete Population Growth

- More appropriate to model populations of plants, insects, mammals, and other organisms that reproduce seasonally.
- Individuals in populations comprise a series of **cohorts** whose members are the same age.

Cohorts

- Time interval begins with appearance of newborns (e.g. spring)
- Surviving individuals produce another cohort of offspring at the beginning of the next interval (e.g. fall)
- Parents may survive to reproduce again

Finite Difference Equation

- Assume a periodic census to quantify **discrete** changes in the population size
- The difference equation projects population size over a finite, observable time interval from the birth of one cohort until the birth of the next.

Parameters

N_t	is population size at time t
b	births per individual during one time interval
p	The individual's probability of surviving from one census to the next

Population Projection

- Population projection over the interval

time = t – 1   to   time = t

$$N_t = pN_{t-1} + pbN_{t-1}$$

Population at time t

An individual's probability of surviving multiplied by the known population in the last time interval: gives the number of individuals who survived.

Plus the new births for those individuals that survived

Creating the Difference Equation

This can be rewritten as:

$$N_t = (p + pb)N_{t-1}$$

New Parameter λ

We can define a new parameter λ (lower-case Greek character lambda).

$$\lambda = (p + pb)$$

λ is the number of survivors and progeny derived from each individual that appeared in the census at the beginning of the interval.

λ is the discrete- or finite-growth rate, but more properly, it is a multiplicative growth factor that determines the proportional change in population size over a discrete interval.

More About λ

- When $\lambda = 1$
  - individuals just manage to replace themselves and population size at the next census remains constant.
- When $\lambda < 1$
  - The population shrinks toward extinction
- When $\lambda > 1$
  - The population grows larger

Population Projection

$$N_t = \lambda N_{t-1}$$

- To project population size backward or forward over one or more time intervals:

$$N_t = \lambda N_{t-1} = \lambda(\lambda N_{t-2}) = \lambda(\lambda(\lambda N_{t-3})) = \ldots$$

$$\boxed{N_t = \lambda^t N_0}$$

from

Populus

Simulations of Population Biology

Don Alstad

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Programming by

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