Density Dependent Population Growth

The logistic model

Assumptions

- Immigration and emigration balance
  - Leaving birth and death as the only rate parameters
- All individuals are identical
- The population is asexual
- Density dependent feedback exerts its effect on population growth instantaneously.

Assumptions (cont.)

Resources are not infinite in the **density *dependent*** model as they were in the **density *independent*** model.

Parameters

***N*(*t*)**	is population size: a continuous function of time, *t*
r	(births – deaths)
K	carrying capacity

Carrying Capacity (*K*)

- Quantified in units of individual population members.
- Rate of change of the population (dN/dt) must decline as the population size (N) approaches the carrying capacity (K).
- When $N = K$, population size has reached the carrying capacity and resource limitation should prevent further population growth.
- When $N < K$, supplies have not been exhausted and population continues to grow.

More about K

- $K - N$ gives the unused capacity of the habitat.

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63 - 20 = 43

- $\left(\dfrac{K - N}{K}\right)$ gives the unused fraction of carrying capacity ranging from 0 (when $N = K$) to 1 (when $N = 0$)

$$\frac{63 - 20}{63} = \frac{43}{63} \approx 0.68$$

The Logistic Model Equation

- Start with density-independent exponential growth equation:

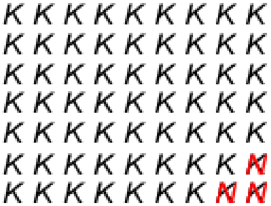
$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN$$

- Now add a term that multiplies the right side of the equation by the unused carrying capacity:

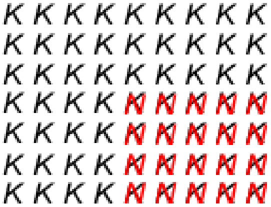
$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(\frac{K - N}{K}\right)$$

How Parameter Changes Affect the Rate of Change of the Population

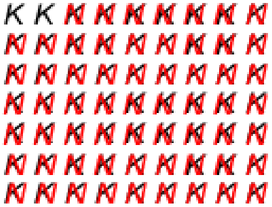
- As N gets closer in value to K



$$rN\left(\frac{K-N}{K}\right)=\frac{63-3}{63}rN\approx 0.95rN$$



$$rN\left(\frac{K-N}{K}\right)=\frac{63-20}{63}rN\approx 0.68rN$$



$$rN\left(\frac{K-N}{K}\right)=\frac{63-61}{63}rN\approx 0.03rN$$

$\frac{\mathrm{d}N}{\mathrm{d}t}$ gets smaller

When $K = N$

- The rate of change is zero when $K = N$.

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right) =$$

$$\frac{63-63}{63} rN =$$

$$0 * rN =$$

$$0$$

Equation to Show Rate of Change

for Values Between $N = K$ and $N = 0$

- The equation $\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(\frac{K-N}{K}\right)$ can be rearranged into the form $y = mx + b$

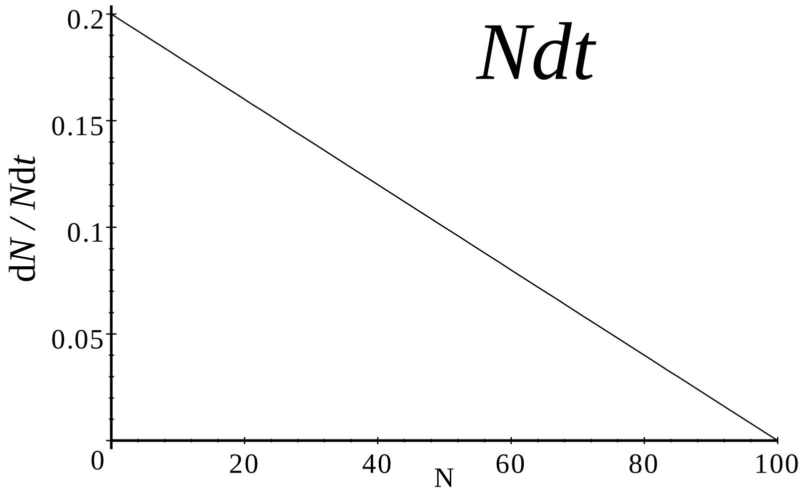
$$\frac{\mathrm{d}N}{N\mathrm{d}t} = r\left(\frac{K-N}{K}\right) \Rightarrow r\left(\frac{1}{K}(K-N)\right) \Rightarrow \frac{r}{K}(K-N)$$

$$\Rightarrow \frac{rK}{K} - \frac{rN}{K} \Rightarrow \boxed{\frac{dN}{Ndt} = -\frac{r}{K}N + r}$$

The *Per Capita* Population Growth Rate

- Is the rate of change per capita and is linear
- Is a declining function of N
- The y intercept is r
- The x intercept is K
- The slope is $-r/K$

$$\frac{dN}{Ndt}$$

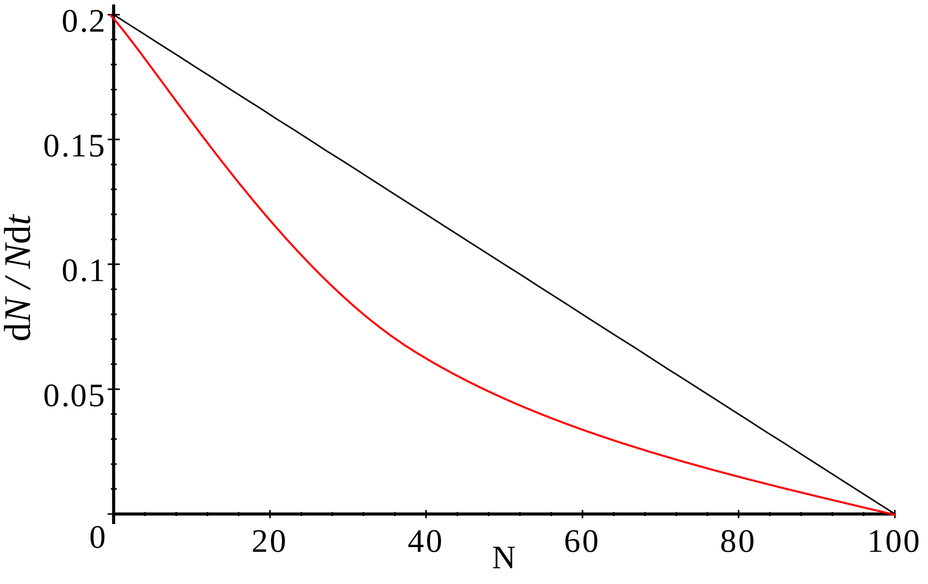


$$\frac{dN}{Ndt} = -\frac{r}{K}N + r$$

Graph for $r = 0.20$ and $K = 100$

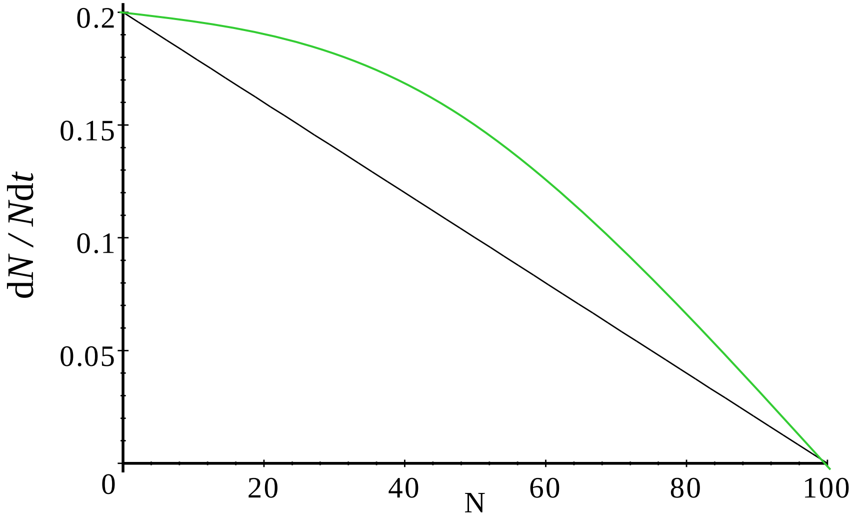
The Effect of Interference Competition on *Per Capita* Population Growth Rate

If individuals hoard resources so that they interfere with their neighbors beyond the extent dictated by basic needs, their competitive effects would be stronger at low density and the graph of the function might look like this ⇨



How Territories of Fixed Size Affect *Per Capita* Population Growth Rate

- Here competitive effects are weak until population density rises to the point where all potential territories fill, as is the case with territorial animals.



Per Capita Versus Population Growth Rates

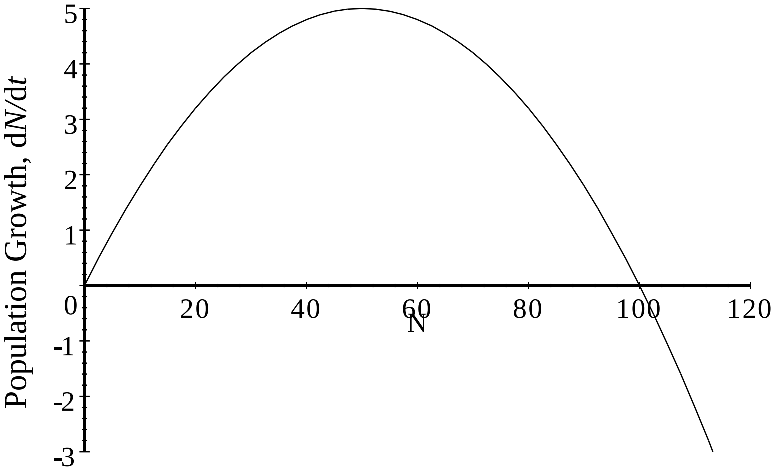
- To find population growth rates rearrange the equation again. Multiply through by N to get:

$$N\left(\frac{dN}{Ndt} = -\frac{r}{K}N + r\right) \Rightarrow \frac{dN}{dt} = rN - \frac{rN^2}{K}$$

Population Growth Rates

Population growth rate, d*N*/d*t*, is a **quadratic** function of *N* with roots (where d*N*/d*t* = 0) at *N* = 0 and *N* = *K*

$$\frac{dN}{dt} = rN - \frac{rN^2}{K}$$



Graph for r = 0.20 and K = 100

Integrating the Rate of Change Equation

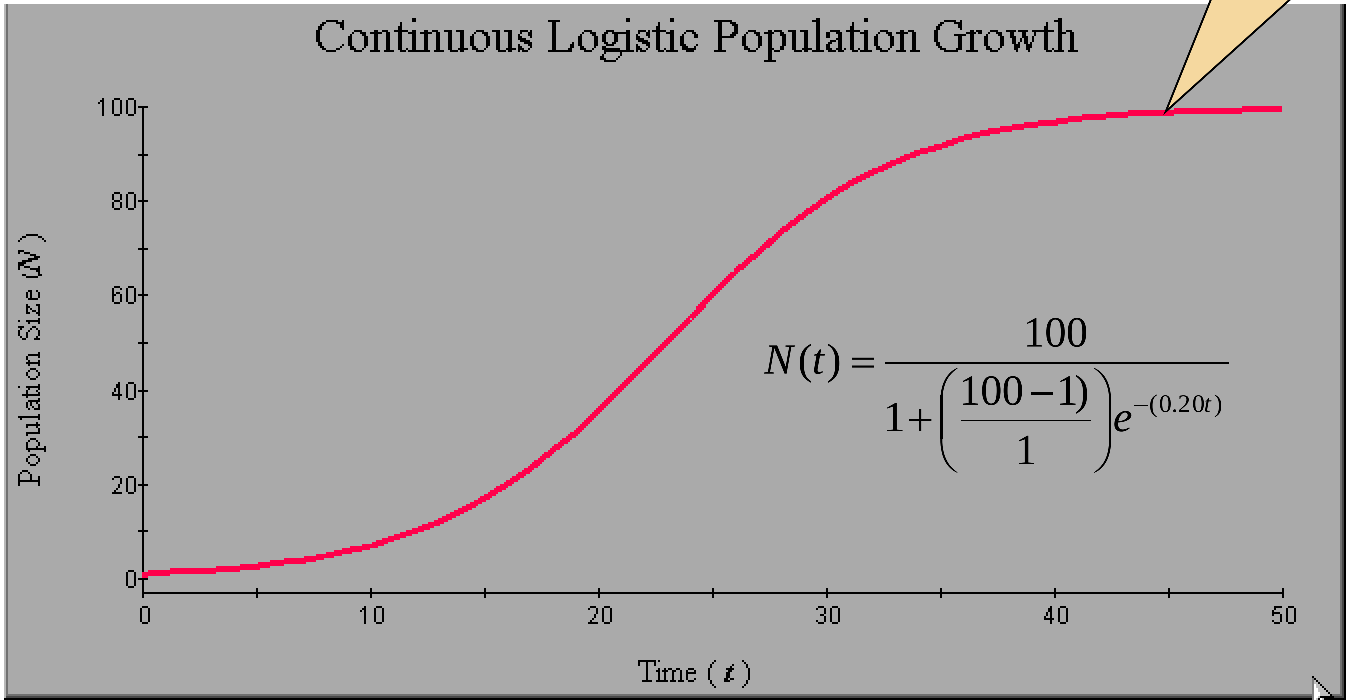
- IS SCARY! There are references in the *POPULUS* help files for those interested in the details.
- Integration gives the equation:

$$N(t) = \frac{K}{1 + \left(\frac{K - N(0)}{N(0)}\right) e^{-(rt)}}$$

Understanding the Continuous Logistic Population Growth Equation

- Let $K = 100$
- Let the starting population $N(0) = 1$
- Let r = 0.2

Often called "sigmoidal" curve



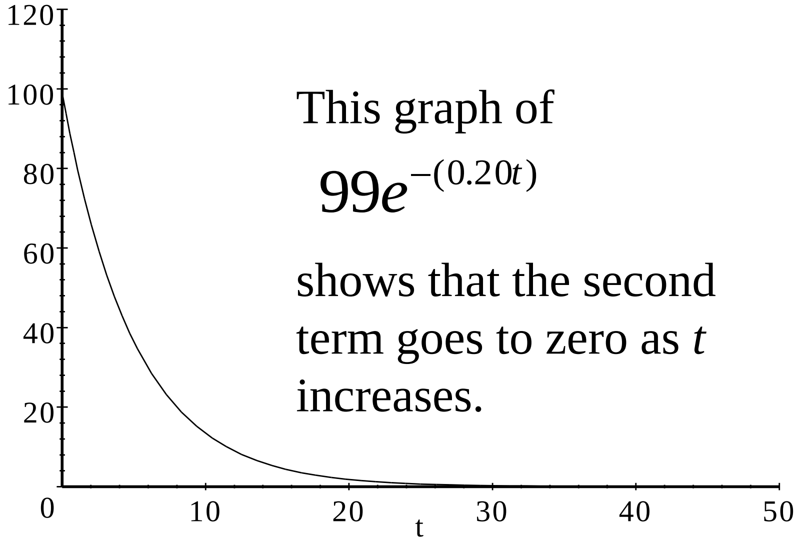
Dissecting the Equation

$$N(t) = \frac{K}{1 + \left(\frac{K - N(0)}{N(0)}\right) e^{-(rt)}}$$

$K = 100$, $N(0) = 1$, $r = 0.2$

$$N(t) = \frac{100}{1 + \left(\frac{100 - 1)}{1}\right) e^{-(0.20t)}}$$

When this part of the equation approaches zero, the denominator approaches 1 and $N(t)$ approaches K, the numerator, 100 in our example.



This graph of

$$99e^{-(0.20t)}$$

shows that the second term goes to zero as t increases.

Dissecting the Equation --Continued

$$N(t) = \frac{K}{1 + \left(\frac{K - N(0)}{N(0)}\right) e^{-(rt)}}$$

$K = 100$, $N(0) = 1$, $r = 0.2$

$$N(t) = \frac{100}{1 + \left(\frac{100 - 100)}{100}\right) e^{-(0.20t)}}$$

$N(t)$ also approaches K (carrying capacity) when $N(0) = K$

$$N(t) = \frac{100}{1 + 0e^{-(0.20t)}}$$

Question One

1. Investigate the effect of initial population size on continuous logistic population growth. Use *POPULUS* to simulate population growth when $N(0) = K$, $N(0) < K$, $N(0) > K$. Describe the curves that result from these simulations.

Question Two

2. Intuitively, how should increasing r affect logistic population growth? Test the prediction you made with *POPULUS* simulations, running continuous logistic cases with $r = 0, 0.5$, and 5, holding $N(0)$ and K constant.

from

Populus

Simulations of Population Biology

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